Comparison of regimes of policies for urban development: a social welfare approach

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Abstract: Tit for Tat is normally conceived of as being a winner that enhances the personal well-being among interactive strategies in the setting of repeated prisoner's dilemma games. Here we prove deductively that, under some conditions, Tit for Tat also outperforms other commonly adopted strategies in terms of enhancing the social welfare, the total of the personal well-beings. The implication is that we might want to seek better interactive strategies, or policies, that contribute most not only to the personal well-being, but also to the social welfare as well. Explanations can be drawn from this analysis on why zoning of a land control measure gives rise to mixed use in urban development.

Keywords: game theory; Tit for Tat; TFT; prisoner's dilemma; policies; zoning.

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1 Introduction

Policies narrowly defined as if-then rules are considered as one of the decision mechanisms of how plans work that affect urban development (Hopkins, 2001). In a strategic setting, such rules could be formulated and explored through games as manifested by the interaction between a local government and developers (e.g., Knaap et al., 1998). The game-theoretic frameworks become inextricably complex in the case of many interacting players, so we focus here on two-person, iterated prisoner's dilemma games for simplicity. A two-person, iterated prisoner's dilemma game contains two players with strategic behaviour in a game played more than once and formulated as a prisoner's dilemma as will be depicted in Section 2. Such a simplification can help clarify doubts on appropriate strategies without losing much generality. Ever since Tit for Tat (TFT) won both the tournaments designed by Axelrod (1984), the interactive strategy in the two-person, iterated prisoner's dilemma game has received much attention in the research of strategic behaviour in many fields (e.g., Stephens et al., 2002; Nowak et al., 2004; McNamara et al., 2004; Nowak and Sigmund, 2005; Rockenbach and Milinsku, 2006). A player adopting the TFT strategy cooperates first and then mimics the strategic behaviour of the other player in the ensuing rounds of the prisoner's dilemma game. Most of such work focuses either on designing better strategies that outperform TFT (e.g., Nowak and Sigmund, 1993; Imhof et al., 2007), or on seeking the conditions under which cooperation between the two players would emerge (e.g., Nowak, 2006). In either research, the focus is placed on the individual player's payoffs, rather than on the overall payoff across the individual players, or social welfare. Intuitively, players who are self-interested seeking their maximum personal payoffs may not necessarily yield a higher payoff for the society as a whole. On the contrary, building on earlier work (Chiu and Lai, 2008), in the present paper we show deductively that, among four commonly noted interactive strategies, namely, always defect (AD), always cooperate (AC), TFT, and random (as will be defined shortly), given some behavioural constraints, TFT results in the highest over payoff of the two-person, iterated prisoner's dilemma game. The reader is encouraged to consult Chiu and Lai's (2008) exposition about the relationship between plans and policies. The model proposed here can be used to depict why mixed use in land comes about in Asian cities (e.g., Lai and Han, 2012). In Section 2, we depict the model of two-person, iterated prisoner's dilemma game. In Section 3, we compare deductively four commonly adopted interactive strategies in the model. In Section 4, we explain some implications of the model on urban development. In Section 5, we conclude.

2 The model

The standard two-person, iterated prisoner's dilemma game modelled here is built on the following payoff table of Table 1 (Axelrod, 1984).

Table 1 The standard payoff table for the two-person, iterated prisoner's dilemma game

	Cooperate	*Defect*
Cooperate	(R, R)	(S, T)
Defect	(T, S)	(P, P)

The payoffs received by players 1 and 2 taking a particular combination of actions are denoted as (X, Y), where X is the payoff received by player 1 adopting the action in the corresponding row, whereas Y is the payoff received by player 2 adopting the action in the corresponding column. Each player can either cooperate or defect, the meanings of which depend on the contexts in which the game is constructed. For example, if player 1 could be the local government and player 2 the developer, 'cooperate' or 'defect' for the local government might mean to provide infrastructure for land development and the developer might respond with cooperation to participate or defection not to participate. If both the local government and the developer cooperate so that the cost of the infrastructure is shared for land development, the outcome results in the payoffs of (R, R), which is better than (P, P) in which no infrastructure is provided resulting in no development. However, if either player defects while the other player cooperates, the defector is the free rider by enjoying the public service without paying for it and thus gets the highest payoff of T, whereas the co-operator bears the total cost of the infrastructure and receives the lowest payoff of S. The logic of the two-person, iterated prisoner's dilemma game requires that $T > R > P > S$, so that the Nash equilibrium of the game if played once is for both players to defect. At Nash equilibrium, no player can move independently to increase his or her payoff, which is thus the dominant strategy (McConnell et al., 2012). The other requirement is that for the game to be played repeatedly, $R > (T + S) / 2$. Otherwise, if both players coordinating to alternate between cooperation and defection would lead to a higher payoff than mutual cooperation, violating the first requirement. It can be easily shown that there are indefinite combinations of T, R, P, and S that satisfy the two requirements, making it difficult to compare deductively how different interactive strategies perform in terms of cumulative payoffs over the iterations.

Here we provide an exact comparison based on a simplified version of the payoff table as shown in Table 2 (Nowak and May, 1993). Note that $1 < b < 2$.

Table 2 The simplified payoff table for the two-person, iterated prisoner's dilemma game

	Cooperate	*Defect*
Cooperate	(1, 1)	(0, b)
Defect	(b, 0)	(0, 0)

Compared to Table 1, since $b > 1$, a closer examination shows that the Nash equilibrium of the two-person, iterated prisoner's dilemma game derived from the payoff table as shown in Table 2 is for both the two players to defect, though $P = S = 0$. In addition, since $b < 2$ so that $R = 1 > (T + S) / 2 = (0 + b) / 2$. The two requirements being satisfied, the logic of the two-person, iterated prisoner's dilemma game is thus retained. The beauty of this simplified version of the two-person, iterated prisoner's dilemma game lies in its simplicity of reducing the analytic structure to a single parameter b, making the exact comparison of different interactive strategies possible.

3 The exact comparison

Four commonly adopted interactive strategies are compared deductively based on the payoff table depicted in Table 2, namely, AD, AC, TFT, and random action (RA). As the label expresses itself, AD (AC) indicates that the player adopting this strategy will take the action of defection (cooperation) no matter what action the other player takes. In particular, for AD, player 1 cooperates initially with a probability of one, and will keep defecting once player 2 defects. If player two defects in iteration 1, then player 1 will defect in return in iteration two with a probability of one. Starting from iteration 2, player 2, noticing that player 1 defected in iteration 1 and to avoid being exploited, will also defect in the ensuing iterations, and the process enters into a punishment phase where both players defect (Dixit and Skeath, 2002). The player adopting the TFT strategy will cooperate in the first iteration, and then depending on the other player's action, the TFT player will copy that action in the ensuing iterations. The RA player simply cooperates or defects in a random fashion, regardless of what action the other player takes. Elsewhere (Chiu and Lai, 2008), we have shown that TFT is the best strategy yielding the highest expected payoff for the *individual* player who adopts that strategy among all the four strategies, for limited and unlimited numbers of iterations. However, we show here, under some conditions, that TFT is also the best strategy yielding the highest *overall* expected payoff across the two players, or the social welfare, for limited and unlimited numbers of iterations.

The four strategies can be represented as game trees. Consider the game tree for player 1 adopting TFT as shown in Figure 1. The thick branch indicates the evolution of possible paths throughout the game tree. In the beginning of iteration 1, player 1 cooperates first and player 2, whose strategies are unknown, can either cooperate or defect. In iteration 2, if player 2 cooperated in the previous iteration, then player 1 will cooperate and player 2 can either cooperate or defect. On the other hand, if player 2 defected in iteration 1, then player 1 will defect and player 2 can either cooperate or defect in iteration 2. This logic applies to the ensuing iterations. All the other three strategies can be represented as the game trees. Note that this game tree can also be used to represent player 2's payoffs if we replace the payoffs at the leaves with those from the player 2's perspective.

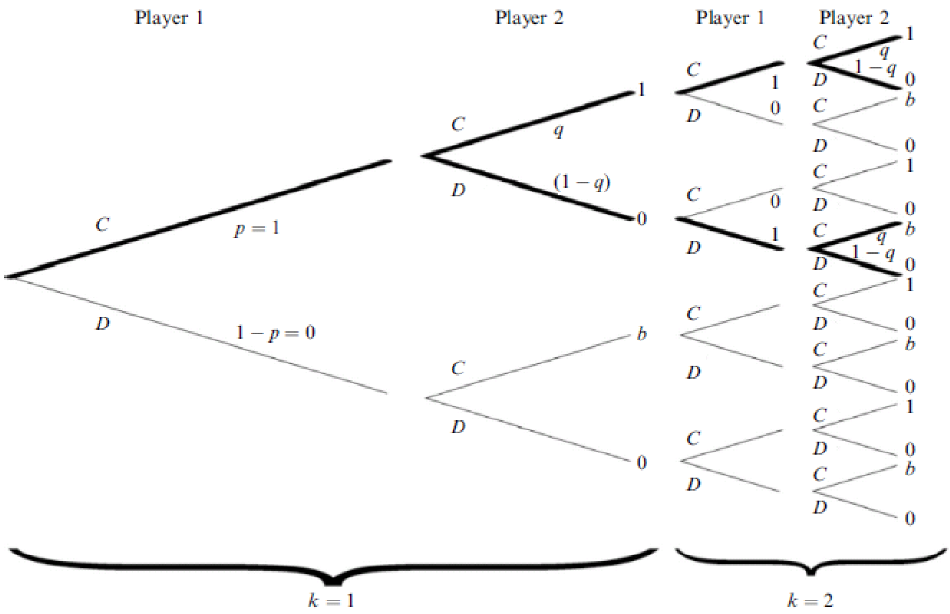
What is remarkable is that all the four strategies are recursive in the game trees, which makes the calculation of the overall expected payoffs tractable. More specifically, let T_k denote a sub-tree in the TFT game tree for iteration k. Referring to Figure 1, a closer examination shows that the following recursive expected payoff functions for both players 1 and 2 obtain:

$$T_3 = qT_2 + (1-q)T_2 = T_2,$$
$$T_4 = qT_3 + (1-q)T_3 = T_3,$$
$$T_n = qT_{n-1} + (1-q)T_{n-1} = T_{n-1}$$

Therefore, for TFT, $T_n = T_{n-1} = T_{n-2} = \ldots = T_3 = T_2$. Since the payoffs for players 1 and 2 in T_1 are q and $q + b - bq$ respectively and for those in T_2 are both $q^2 + qb - q^2b$, it can be easily shown that the sum of the overall expected payoffs across the two players up to iteration $k = n$ is:

$$S_{tt} = (2q + b - bq) + 2(n-1)\left(q^2 + bq - bq^2\right) \tag{1}$$

Figure 1 The game tree for player 1 adopting TFT for iterations $k = 1$ and 2



C = cooperates; D = defects; p is the probability player one will cooperate and q the probability that player two will cooperate; b is a payoff value < 2 and > 1.

Let P_k denote a sub-tree in the game tree of AD for iteration k. The recursive expected payoff functions for both players 1 and 2 are:

$$P_2 = qP_1 + (1-q)0 = qP_1,$$
$$P_3 = qP_2 + (1-q)0 = qP_2,$$
$$P_n = qP_{n-1} + (1-q)0 = qP_{n-1}$$

Since the payoffs for players 1 and 2 in P_1 are q and $q + b - qb$ respectively, we have the sum of the overall expected payoffs across the two players up to iteration $k = n$ as:

$$S_{pt} = (2q + b - bq)\frac{(1-q^n)}{1-q} \tag{2}$$

Let F_k denote a sub-tree in the game tree of AC for iteration k. The recursive expected payoff functions for both players 1 and 2 are:

$$F_2 = qF_1 + (1-q)F_1 = F_1,$$
$$F_3 = qF_2 + (1-q)F_2 = F_2,$$
$$F_n = qF_{n-1} + (1-q)F_{n-1} = F_{n-1}$$

Therefore, for AC, $F_n = F_{n-1} = F_{n-2} = \ldots = F_3 = F_2 = F_1$. Since the payoffs for players 1 and 2 in F_1 are q and $q + b - qb$ respectively, we have the sum of the overall expected payoffs across the two players up to iteration $k = n$ as:

$$S_{ft} = n(2q + b - qb) \tag{3}$$

Let M_k denote a sub-tree in the game tree of RA for iteration k. The recursive expected payoff functions for both players 1 and 2 are:

$$M_2 = pqM_1 + p(1-q)M_1 + (1-p)qM_1 + (1-p)(1-q)M_1 = M_1$$

Thus we have $M_n = M_{n-1} = M_{n-2} = \ldots = M_3 = M_2 = M_1$. Since the payoffs for players 1 and 2 in M_1 are the same as $pq + pb - pbq$, the sum of the overall expected payoffs across the two players up to iteration $k = n$ becomes:

$$S_{mt} = 2n(pq + pb - pbq) \tag{4}$$

Given the general forms of the overall expected payoff across the two players in the two-person, iterated prisoner's dilemma game, we are able to compare deductively which interactive strategy performs the best in terms of the overall expected payoff. This is done in two cases: limited and unlimited numbers of iterations.

3.1 *The case of a limited number of iterations*

Consider first the comparison between TFT and AD in the case of a limited number of iterations. Referring to equations (1) and (2), because we are interested in the *repeated* two-person prisoner's dilemma game, we have for $k = 2$:

$$S_{tt} - S_{pt} = \left[(2q + b - bq) + 2\left(q^2 + bq - bq^2\right)\right] - (2q + b - bq)\frac{\left(1-q^2\right)}{1-q} = bq - bq^2 = bq(1-q) > 0$$

Assume that $(S_{tt} - S_{pt}) > 0$ for $k = n$, that is:

$$\left[(2q + b - bq) + 2(n-1)\left(q^2 + bq - bq^2\right)\right] - (2q + b - bq)\frac{\left(1-q^n\right)}{1-q} > 0 \tag{5}$$

when $k = n + 1$, we have:

$$S_{tt} - S_{pt} = \left[(2q + b - bq) + 2n\left(q^2 + bq - bq^2\right)\right] - (2q + b - bq)\frac{\left(1-q^{n+1}\right)}{1-q}$$

$$= \left[(2q + b - bq) + 2(n-1)\left(q^2 + bq - bq^2\right) + 2\left(q^2 + bq - bq^2\right)\right] - \left[(2q + b - bq)\frac{\left(1-q^n\right)}{1-q} + \left(2q^n + bq^{n-1} - bq^n\right)\right]$$

$$= \left[(2q + b - bq) + 2(n-1)\left(q^2 + bq - bq^2\right) - (2q + b - bq)\frac{\left(1 - q^n\right)}{1 - q} \right]$$
$$+ \left[2\left(q^2 + bq + bq^2\right) - \left(2q^n + bq^{n-1} - bq^n\right) \right]$$
$$= \left[(2q + b - bq) + 2(n-1)\left(q^2 + bq - bq^2\right) - (2q + b - bq)\frac{\left(1 - q^n\right)}{1 - q} \right]$$
$$+ \left[2q^2\left(1 - q^{n-2}\right) + \left(2 - q^{n-2}\right)bq(1 - q) > 0 \right]$$

Based on the logic of induction, we can conclude that TFT is better than AD for $k \geq 2$, in terms of the overall expected payoffs across the two players.

We now turn to the comparison between TFT and AC. Referring to equation (1) and equation (3), when $k = 2$ we have:

$$\begin{aligned} S_{tt} - S_{ft} &= \left[(2q + b - bq) + 2\left(q^2 + bq - bq^2\right) \right] \\ &\quad -2(2q + b - qb) = 2\left(q^2 + bq - bq^2\right) \\ &\quad -(2q + b - bq) = (1 - q)(2bq - b - 2q) \end{aligned}$$

whether $(2bq - b - 2q) > 0$ depends on the probability q and a closer examination shows that for this inequality holds, $2q > \frac{b}{b-1} > 1,$ where $1 < b < 2$.

Assume that this inequality holds and that when $k = n$, $S_{tt} - S_{ft} > 0$, that is:

$$\begin{aligned} &\left[(2q + b - bq) + 2(n-1)\left(q^2\ \ bq - bq^2\right) \right] \\ &-n(2q + b - qb) = 2(n-1)\left(q2 + bq - bq^2\right) \\ &-(n-1)(2q + b - qb) > 0 \end{aligned} \tag{6}$$

Let $k = n + 1$ and assume that $2q > \frac{b}{b-1} > 1,$ and we have:

$$\begin{aligned} S_{tt} - S_{ft} &= 2n\left(q^2 + bq - bq^2\right) - n(2q + b - qb) \\ &= \left[2(n-1)\left(q^2 + bq - bq^2\right) - (n-1)(2q + b - qb) \right] \\ &\quad + \left[2\left(q^2 + bq - bq^2\right) - (2q + b - qb) \right] \\ &= \left[2(n-1)\left(q^2 + bq - bq^2\right) - (n-1)(2q + b - qb) \right] \\ &\quad + (1 - q)(2bq - b - 2q) > 0 \end{aligned}$$

Based on the logic of induction, we can conclude that TFT is better than AC for $k \geq 2$, in terms of the overall expected payoffs across the two players, but under the condition that $2q > \frac{b}{b-1} > 1$.

Finally, we compare TFT with RA. Referring to equations (1) and (4), when $k = 2$, we have:

$$
\begin{aligned}
S_{tt} - S_{mt} &= \left[(2q+b-bq)+2\left(q^2+bq-bq^2\right)\right] \\
&\quad -4(pq+pb-pbq) \\
&= 2q+b+2q^2+bp-2bq^2-4pq-4pb+4pbq \\
&= (2q-4p+1)[b(1-q)+q]+q.
\end{aligned}
$$

thus

$$(2q-4p+1) > \frac{-q}{b(1-q)+q},$$

or

$$(q-2p) > -\left\{\frac{q}{2\left[b(1-q)+q\right]}+1\right\},$$

Then TFT is better than RA.

Now, assume when $k = n$ and the following inequality holds:

$$S_{tt} - S_{ft} = \left[(2q+b-bq)+2(n-1)\left(q^2+bq-bq^2\right)\right]-2n(pq+pb-pbq) > 0 \tag{7}$$

Let $k = n + 1$, we have:

$$
\begin{aligned}
S_{tt} - S_{ft} &= \left[(2q+b-bq)+2(n-1)\left(q^2+bq-bq^2\right)\right] \\
&\quad -2n(pq+pb-pbq)+2\left(q^2+bq-bq^2\right)-2(pq+pb-pbq) > 0, \\
&\quad \text{because } 2\left(q^2+bq-bq^2\right)-2(pq+pb-pbq) \\
&\quad = 2(q-p)[b(1-q)+q] > 0 \text{ if } (q-p) > 0.
\end{aligned}
$$

Based on the logic of induction, we can conclude that TFT is better than RA for $k \geq 2$, in terms of the overall expected payoffs across the two players, but under the condition that $(q - p) > 0$.

3.2 *The case of an unlimited number of iterations*

For the comparison between TFT and AD, when $k = n$ we know that the difference of the overall expected payoffs under the two strategies is given in Inequality (5). When $n \to \infty$, implying an unlimited number of iterations, the overall expected payoff difference between TFT and AD is expressed as:

$$\lim_{n\to\infty}\left\{\left[(2q+b-bq)+2(n-1)\left(q^2+bq-bq^2\right)\right]-(2q+b-bq)\frac{\left(1-q^n\right)}{1-q}\right\}$$

$$\lim_{n\to\infty}\left\{\left[(2q+b-bq)+2(n-1)q\left(q+bq-bq\right)\right]-(2q+b-bq)\frac{\left(1-q^n\right)}{1-q}\right\}$$

$$= \infty-(2q+b-bq)\frac{q}{1-q} > 0$$

We can conclude that TFT yields the higher overall expected payoff across the two players than AD in the case of an unlimited number of iterations.

For the comparison between TFT and AC, the difference of the overall expected payoffs across the two players is given in Inequality (6). When $n \to \infty$, the overall expected payoff difference between TFT and AC becomes:

$$\lim_{n\to\infty}\left\{\left[(2q+b-bq)+2(n-1)\left(q^2+bq-bq^2\right)\right]-n(2q+b-bq)\right\}$$
$$=\lim_{n\to\infty}\left\{(n-1)\left[2\left(q^2+bq-bq^2\right)-\left(2q+b-qb\right)\right]\right\}$$
$$=\lim_{n\to\infty}[(n-1)(1-q)(2bq-b-2q)]$$

For $(2bq - b - 2q) > 0$, we have $2q > \frac{b}{b-1} > 1$. Therefore, TFT is better than AC if this inequality holds.

Finally, we compare TFT with RA. The difference of the overall expected payoffs across the two players is shown in Inequality (7). When $n \to \infty$, the overall expected payoff difference between TFT and RA is shown as:

$$\lim_{n\to\infty}\left\{\left[(2q+b-bq)+2(n-1)\left(q^2+bq-bq^2\right)\right]-2n(pq+pb-pbq)\right\}$$
$$=\lim_{n\to\infty}\left\{(2n-1)(q-p)\left[b(1-q)+q\right]+q\right\}$$

Thus, if $(q - p) > 0$, then TFT is better than RA.

It is clear that the answer really depends to the question of whether TFT outperforms the other tree strategies, namely, AD, AC, and RA, in terms of the overall expected payoffs across the two players in the two-person, iterated prisoner's dilemma game. More specifically, TFT unconditionally outperforms AD. But for AC and RA, if:

$$2q > \frac{b}{b-1} > 1$$

and

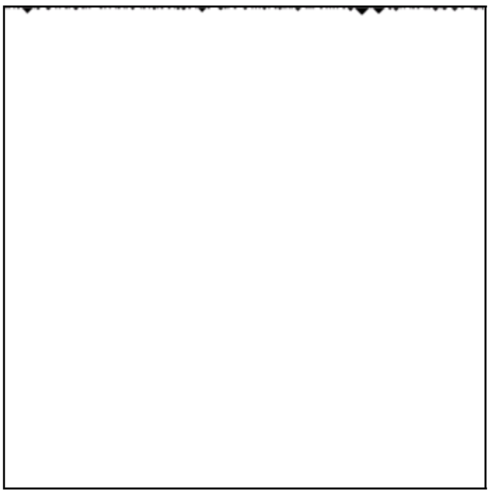
$$(q-2p) > -\left\{\frac{q}{2[b(1-q)+q]}+1\right\}$$

respectively for the case of a limited number of iterations, and if $2q > \frac{b}{b-1} > 1$ and $(q - p) > 0$ respectively for the case of an unlimited number of iterations, TFT still performs the best. In other words, in order for TFT to outperform AC, $q > \frac{1}{2}$, meaning that player 2 is inclined to cooperate. For RA, the implication for the probability distribution of p and q for the two players is difficult to derive in the case of a limited number of iterations. However, in the case of an unlimited number of iterations, the condition that TFT outperforms RA implies that player 2 is more inclined to cooperate than player 1.

4 Some implications on urban development

The game-theoretic analysis exposited in the present paper can be used to draw implications on urban development. For example, mixed use may occur regardless of whether a zoning system for single use is imposed on a city. In particular, we argue that as long as the zoning system allows to a limited extent mixed use, as the cases of Taiwan and Japan, the system would result in fractal rather than Euclidean structural morphology. Firstly, according to Schelling (2006), segregation of neighbourhoods emerges when there is mild prejudice against different ethnic groups. On the other hand, we argue that if there is no discrimination against mixed use, which would be the case in most Asian cities, segregation in land use would not occur. Secondly, mixed use is a more natural development pattern that might yield higher target profits than single use because of increasing returns to complementarity of uses. Compare mixed use and single use patterns of commercial and residential developments. In the mixed use pattern, the adjacency between commercial and residential uses would result in higher property right capture that is left in the public domain as manifested by potential gains due to better accessibility between the two uses than when they are segregated, thus resulting in the total profits of the mutual development, which could be formulated as a prisoner's dilemma game as shown below.

Figure 2 The space-time plot of the rule of 10000000

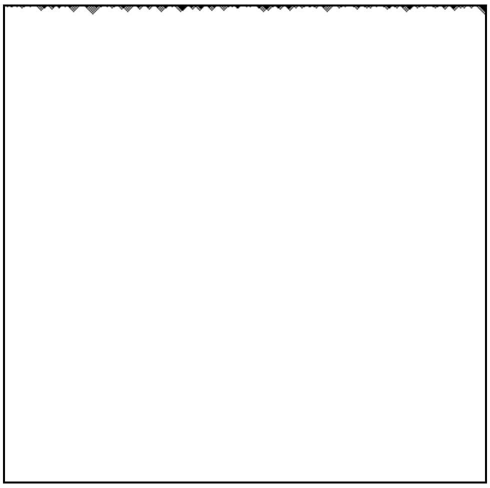


Consider only two land uses, commercial and residential, which could be adopted by two developers on two neighbouring parcels in a community. The development decisions adopted by the two developers can be represented as a two-person, iterated prisoner's dilemma game in that each developer can either cooperate to develop the land for residential use or defect for commercial use. Nowak and May (1993) design a simplified version of the two-person, iterated prisoner's dilemma game that serves as a basis for our deductive comparison. In their formulation, Nowak and May (1993) reduce the payoff table of the two-person, iterated prisoner's dilemma game into one that contains only one parameter b as follows.

	C	*D*
C	1	0
D	*b*	0

In the above payoff table, the values represent the payoffs received by each player when that player (as shown in the rows) takes a certain action, while the other player (as shown in the columns) takes another action. For example, if player one cooperates and player two also cooperates, then player one will receive a payoff of unity. If player one defects while player two cooperates, then player one will receive a payoff of *b*. For the simplified version of the two-person, iterated prisoner's dilemma game to be equivalent to the original one, b must be greater than unity and less than two, so that the Nash equilibrium settles on the combination of actions where both players would defect.

Figure 3 The space-time plot of the rule of 10100000



Chiu and Lai (2008) compare four strategies in the simplified version of two-person, iterated prisoner's dilemma game, namely, AD/cooperate (AD or AC), TFT, and RA. Intuitively, in the land development context, AD or AC would result in single use and TFT and RA would result in mixed use of spatial pattern. Chiu and Lai (2008) are able to show that in the case of an either limited or unlimited number of iterations, the ranking of strategies in terms of the overall expected payoff obtained by a single player is TFT, RA, AC, and AD, implying that the strategies resulting in the outcome of mixed use are dominant. In addition, in the present paper we show that TFT outperforms conditionally other strategies even when the overall expected payoff is calculated across the two players. Note, however, that the TFT strategy might be difficult to adopt in practice because of the significant cost of revising a development decision, or irreversibility (Hopkins, 2001). Indeed, Hopkins (1979) argues that quadratic models reminiscent of mixed use in plan making is more effective than linear-programming models reminiscent of single-used zoning. Both empirical and simulated data demonstrate that the spatial pattern of land uses in Taipei is fractal and mixed use, rather than Euclidean and single use (Lin and Lai, 1998; Lai and Chen, 2006).

Table 3 Transition rules corresponding to the values of *b*

Rule	*Binary code*	*Wolfram's classes*
b 2/3	10000000	Class 1
2 / 3 < *b* < 3 / 4	10100000	Class 1
3 / 4 *b* < 3 / 2	10100100	Class 2
3 / 2 *b* 5 / 3	11101100	Class 2
b > 5 / 3	11111110	Class 1

Whether single or mixed use pattern would prevail could be determined, on the other hand, by the parameter *b*. This question can be investigated through one-dimensional cellular automata (Wolfram, 2002). Consider an array of cells. In the land development context, suppose that each cell represents a block and there are only two permissible types of land use for each block, residential or commercial. The payoff matrix for the land development is written as follows.

	R	*E*
R	1	0
E	*b*	0

where *R* stands for residential and *E* symbolises commercial with *b* as the parameter in the original payoff table of the two-person, iterated prisoner's dilemma game. The above payoff matrix assumes that the payoff of a block is 1 when both blocks are in residential use and *b* when a block in commercial use interacts with another in residential use. In other situations, the payoff is 0. In the elementary cellular automata with two neighbours, there are a total of eight arrays for the values of a cell and its two neighbours: 111, 110, 101, 100, 011, 010, 001 and 000. When each of the two neighbours interacts with its outward neighbour, there are four possibilities for each array, as shown below. The numbers in the brackets denote the payoffs of three cells in the simplified prisoner's dilemma game, as shown in the above matrix. The last row shows the sum of the payoffs of the four possibilities of each of the three cells arrays.

01110	01100	01010	01000	00110	00100	00010	00000
(*b* 0 *b*)	(*b* *b* 2)	(2*b* 1 2*b*)	(2*b* 2 3)	(2 *b* *b*)	(2 2*b* 2)	(3 2 2*b*)	(3 3 3)
01111	01101	01011	01001	00111	00101	00011	00001
(*b* 0 0)	(*b* *b* 1)	(2*b* 1 *b*)	(2*b* 2 2)	(2 *b* 0)	(2 2*b* 1)	(3 2 *b*)	(3 3 2)
11110	11100	11010	11000	10110	10100	10010	10000
(0 0 *b*)	(0 *b* 2)	(*b* 1 2*b*)	(*b* 2 3)	(1 *b* *b*)	(1 2*b* 2)	(2 2 2*b*)	(2 3 3)
11111	11101	11011	11001	10111	10101	10011	10001
(0 0 0)	(0 *b* 1)	(*b* 1 *b*)	(*b* 2 2)	(1 *b* 0)	(1 2*b* 1)	(2 2 *b*)	(2 3 2)
(2*b* 0 2*b*)	(2*b* 4*b* 6)	(6*b* 4 6*b*)	(6*b* 8 10)	(6 4*b* 2*b*)	(6 8*b* 6)	(10 8 6*b*)	(10 1 10)

It can be easily shown that when the value of *b* falls in different intervals as shown in Table 3, different transition rules reign of the elementary cellular automata and the space-time plots corresponding to these rules show different patterns. According to Wolfram's classification, a Class 1 rule results in homogeneous pattern with all white or black cells (see Figures 2, 3, and 5), reminiscent of a single use neighbourhood, while a Class 2 rule comes up with a pattern of fixed structures (see Figures 4 and 5, reminiscent of a mixed use neighbourhood. Note that a white cell symbolises '0' as residential use and a black cell represents '1' as commercial use.

Figure 4 The space-time plot of the rule of 10100100

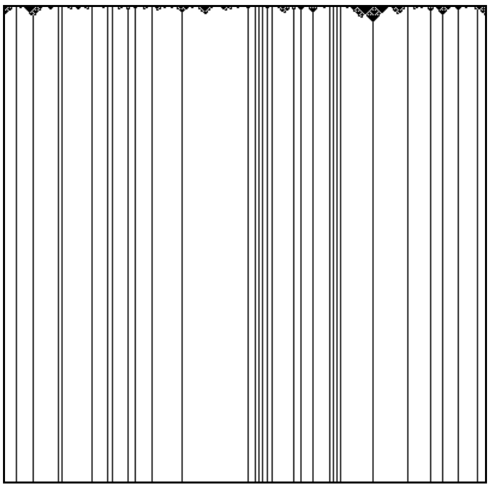


Figure 5 The space-time plot of the rule of 11101100

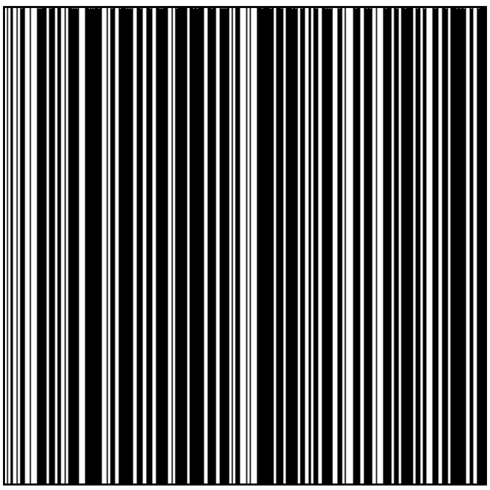


Figure 6 The space-time plot of the rule of 11111110

5 Conclusions

We have conducted an exact deductive comparison among four commonly applied interactive strategies in the two-person, iterated prisoner's dilemma game in terms of the overall expected payoff, or social welfare, across the two players. TFT as conceived normally of as being a winner of the two-person, iterated prisoner's dilemma game from the perspective of the single player adopting that strategy, could potentially contribute the most to the overall expected payoff across the two players. The implication is that we might want to seek better interactive strategies, or policies, that contribute the most not only to the personal well-beings, but also to the social welfare of the society as well. Explanations can be drawn from this analysis on why zoning gives rise to mixed use in urban development. In particular, whether mixed use would emerge depends in part on the value of the parameter *b*, the interactive payoff between different land uses. The model is of course a simple manifestation of the real world urban development process, but it captures to some extent the mechanism underlying that process.

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