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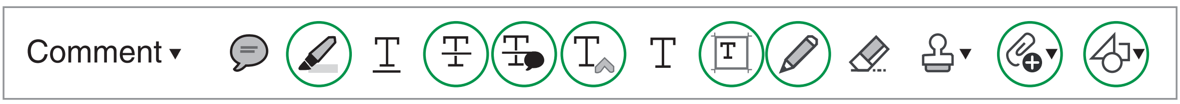

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An anatomy of time explicit planning behavior for urban complexity

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Abstract

Planning has long been perceived as intervention in a complex spatial system that tends toward equilibrium. In this perspective, time is implicit and dynamic details do not matter. As a result, little has been said in the literature about planning behavior that takes into account time and dynamic details. Exploration into planning behavior is important in the face of complex systems that are path dependent and far from equilibrium. The purpose of the present paper is therefore to model normative planning behavior based on Savage's (1954) utility theory, Marschak's (1974) theory of teams, and Hopkins's (1980) definition of plans (i.e. planning is an activity of information gathering and producing to reduce uncertainty), to interpret the planner's behavior on plan making, implementation, and revision.[Per journal style, abstracts should not have reference citations. Therefore, can you kindly delete these reference citations (Savage, 1954; Marschak, 1974; Hopkins, 1980) and rephrase the sentences as appropriate?] This model fits well the emerging perspective of the city in that urban development is non-equilibrium. We first define a simplified planning environment in which there are only one planner and one actor with three worlds: the grand world, the planner's world, and the actor's world, the latter two being small worlds. The notion of small world was first proposed by Savage (1954) and provides a useful way of explaining planning behavior. In the small worlds, the planner and the actor simultaneously select optimal actions among a set in order to maximize their expected utilities. Due to the mathematical property of the small world notion, planning behavior thus defined can be formulated analytically so that the planning process can be depicted in a precise, concrete language. The model proposed in the present paper is normative in nature, emphasizing on how planning behavior should take place and providing insights into how that behavior actually does come about in reality. In its current formulation, the model is only a preliminary approximation of normative planning behavior, but prompts some research questions worth pursuing, such as how multiple planners and actors make and use plans in a more complex situation and what planning procedures would be effective through computer simulations in the face of complexity.

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Planning behavior, iterative procedure, non-equilibrium, urban complexity, utility theory

Introduction

There is a shift in perspective recently about cities from viewing urban development as tending toward equilibrium to considering it as far from equilibrium (e.g. Arthur, 2015; Batty, 2014). This shift in perspective about cities presumably has a significant impact on how we plan for the city. For example, if urban development tends toward equilibrium, then planning is nothing but selecting a target and then solving for and setting up a price system so that the city would eventually evolve toward that target, the details of how the city moves from the status quo to that target being largely ignored (e.g. Hopkins, 1974). On the other hand, if urban development is far from equilibrium, then planning and acting is like canoeing on a river, something that the planner must constantly do in order to reach his/her goal (Hopkins, 2001), implying that the details of the system dynamics matter.

In the present paper, we take the viewpoint that urban development is path dependent and non-equilibrium and that planning is something that must be done constantly. Such a dynamic view of planning is closer to what we observe in reality in which the planner goes about in solving planning problems rather than doing equilibrium analysis. However, there are few analytic works in depicting such planning behavior (exception including Schaeffer and Hopkins, 1987). In order to understand how planning behavior occurs in the context of non-equilibrium urban development and prescribe how such behavior should take place, an axiomatic foundation is important and needed to serve as a starting point. Based on that foundation, it is possible to relax stringent conditions to cope with real, messy psychological traits that affect planning behavior.

The study of planning behavior is still in its early stage, even though its origin can be traced back to the early 1980s (Hopkins, 1980). Schaeffer and Hopkins (1987) first described planning behavior of land developers based on economics of information. Individual developers make their decisions on land development based on discounted present values of expected worth of rights in land taking into account cost of planning. Knaap et al. (1998) elaborate this formulation further by taking into account the role of government and the interaction between the local government and developers using non-cooperative game theory. Still, only the behavior of individual developers, not groups or firms, is modeled in this later framework. However, these pioneering works on planning behavior provide a conceptual basis for the axiomatic approach to planning that we attempts to endeavor here.

The traditional, perhaps most natural, way of attaining knowledge is starting with simple formulations by imposing restrictive assumptions on the system under consideration, and then elaborating from the original formulations by relaxing some or all of the assumptions to fit empirical observations. Therefore, our modeling strategy starts with the subjective expected utility theory, or SEU model, because it is the most complete treatment of rational choice for making a *single* decision. We then expand the SEU model to consider linked, multiple decisions as a plan in the context of non-equilibrium urban development. Some implications are derived from this modeling approach in relation to plan making in the face of complexity. We acknowledge that the SEU model suffers from psychological attacks for being unrealistic (e.g. Kahneman, 2013; Kahneman and Tversky, 1979), but the axiomatic foundation serves as a starting point from which we can gain a better understanding of planning behavior in the context of non-equilibrium urban development.

Following this modeling strategy, in the next section, we depict the theoretical framework of our axiomatic approach to planning. In Section "Interpretation of planning behavior", we interpret planning behavior of making multiple, linked decisions using the theoretical framework. Then we discuss about some implications of plan making in the face of urban complexity derived from our model. We conclude in the final section.

Theoretical framework

There are various versions of subjective expected utility theory (e.g. Arrow, 1979; Savage, 1954; von Neumann and Morgenstern, 1972). In the present paper, we base our theoretical framework on the SEU model proposed by Savage (1954) because that model implicitly incorporates the decision maker's cognitive process and choice behavior. In addition, the notion of small world proposed in that model provides a useful basis on which the interaction takes place between the planner and the actor.

Small world

Before proceeding to depict our model of planning behavior, it is useful to introduce some basic ideas about Savage's notion of small world. Assume that S is a set of descriptions in which the element s depicts an unknown variable of a situation faced by the individual and there corresponds a consequence to each action. Assume also that those elements are mutually independent and exclusive and only one of them can describe the situation correctly. Savage calls those elements possible states of the world. Assume further that C is the set of consequences which are also the states of the world. The elements in C are also mutually independent and exclusive and only one of them obtains. For each element s in S and each action f in the set F_0 (all possible actions), assume that $f(s)$ stands for the correct consequence in C for the individual when situation s occurs. As a result, for each action in F_0, there is a mapping from S to C. Savage calls this pair (S, C) "small world" (Shafer, 1988).

According to Savage, "small world" is a decision situation facing the decision maker which is composed of a set of "states" and a set of "actions". Consequences are a function of actions, and for our modeling purposes, consequences are replaced by actions in the small world notion. States in the small world are the unknowns that will obtain in the near future subject to uncertainty. As a result, the decision situation facing the decision maker is: Under the condition that which state obtains is uncertain, how does the decision maker go about selecting among a set of actions the one in order to maximize his/her expected utility?

In the context of planning, we define small world as a planning situation facing the planner who makes plans and the actor (decision maker) who takes actions accordingly. In the planning situation, there is a set of all possible states of the planner's world and a set of actions to be selected by the planner. However, Savage (1954) does not explain how those actions are constructed, which begs future studies. Our emphasis here is on how the small worlds of the planner's world and the actor's world come about and how they go about making decisions in their respective small worlds.

The planner and the actor each own respective small worlds, and we call the planner's small world the planning world and the actor's small world the decision world. Each of them is faced with a different planning situation because of distinct understandings of the planning problem. In other words, because of the difference in the planner's and the actor's cognitive capabilities of perceiving the planning problem, the elements comprising the small worlds (a set of states and actions) are distinct. As a result, the planning world and the decision world are the transformations of the planning problem into the planner's and the actor's heads of psychological understandings, respectively.

The planner makes plans for the actor to follow. Plan is defined as a set of related, contingent decisions (Hopkins, 1980); therefore, plan making is equivalent to selecting among a set of actions (plans) the one that is optimal in order to maximize the planner's expected utility. In other words, the planner makes decisions in his/her own small world in order to make plans. [AQ1] When new information or unexpected events occur, the planner's or the actor's small world will change, resulting in changes in the planning situation. The planner must then select the action anew, that is, revising the plan. The making, using, and revising plans in the context of non-equilibrium urban development are considered here as planning behavior which can be depicted according to the basic terms introduced.

Primitives

The normative planning behavioral model constructed in the present paper includes some basic primitive ideas other than Savage's small world. First, we identify a simplified planning environment in which there exists two agents, the planner of an individual or a coherent group who makes plans and the actor of an individual or a coherent group who takes action accordingly. The planner and the actor may or may not be the same person or group. Here, we consider the planner and the actor as a two-person team, and according to theory of teams, the two agents have the same goal, that is, they share the same preferential structure about the states of the world (Marschak, 1974).

In Marschak's (1974) theory of organization, a three-phase team is introduced in that the team members make observations, send and receive messages, and perform actions. For each individual in the team, there are three kinds of activities: (1) to make an observation on the external world, (2) to perform an action upon the external world, and (3) to send to a co-member a message (report), i.e. a statement about the external world. A non-negative number will measure the cost of the message. These activities are then described in terms of mathematical terms and the problem is to search for the best rules for action and communication. The criterion of maximizing expected utility is applied for determining the best rules for such a problem.

Two types of problems are distinguished: procedure and network problems. In the former, one seeks to determine what each member has to do or to communicate in response to observations he/she makes and to messages he/she receives, the members from whom he/she can receive and to whom he/she can send messages being given. In the latter, a more complete team problem or a problem of the team's constitution, the network is not given. It is apparent that in the simplified planning environment of one planner and one actor, the network problem is not existent because there are only two members in the team and we assume the two members can exchange information freely.

The simplified planning environment is similar to Marschak's two-person team: an actor and an observer, with more or less costly physical facilities for observation and communication. We assume, however, that the communication is costless. To simplify, we further assume, as in Marschak's assumptions about the two-person team, that the network is given except that the communication between the planner and the actor are two-way communication. That is, the planner (observer) sends messages (report) to the actor and the actor responds also with his/her observation about the small word and can take actions upon it. The planning problem thus becomes a procedural one. That is, we assume that the two agents communicate freely by sending signals to each other so that plans are made, implemented, and revised based on the interaction between the two agents. In the simplified planning environment, besides the roles played by the planner and the actor, there exists the planner's world, the actor's world, and the grand world. The grand world is also composed of a set of states and actions, and the planner's world and the actor's world are small worlds as depicted. The difference between the grand world and a small world is that the states in the former are the universal set of all fundamental states and the states in the small world are subsets of those in the grand world. As a result, a small world is identified in the grand world.

The complete planning process can be divided into *m* repetitive time intervals (*m* is not fixed depending on the planner's and the decision actor's resources or satisfaction level) (cf. Figure 1), each time interval standing for an implementation/revision of plans. There can be two reasons for plan revision: (1) the actor is unsatisfied with the planner's plan and (2) new information occurs or unexpected events happen. Therefore, the length of the iterative

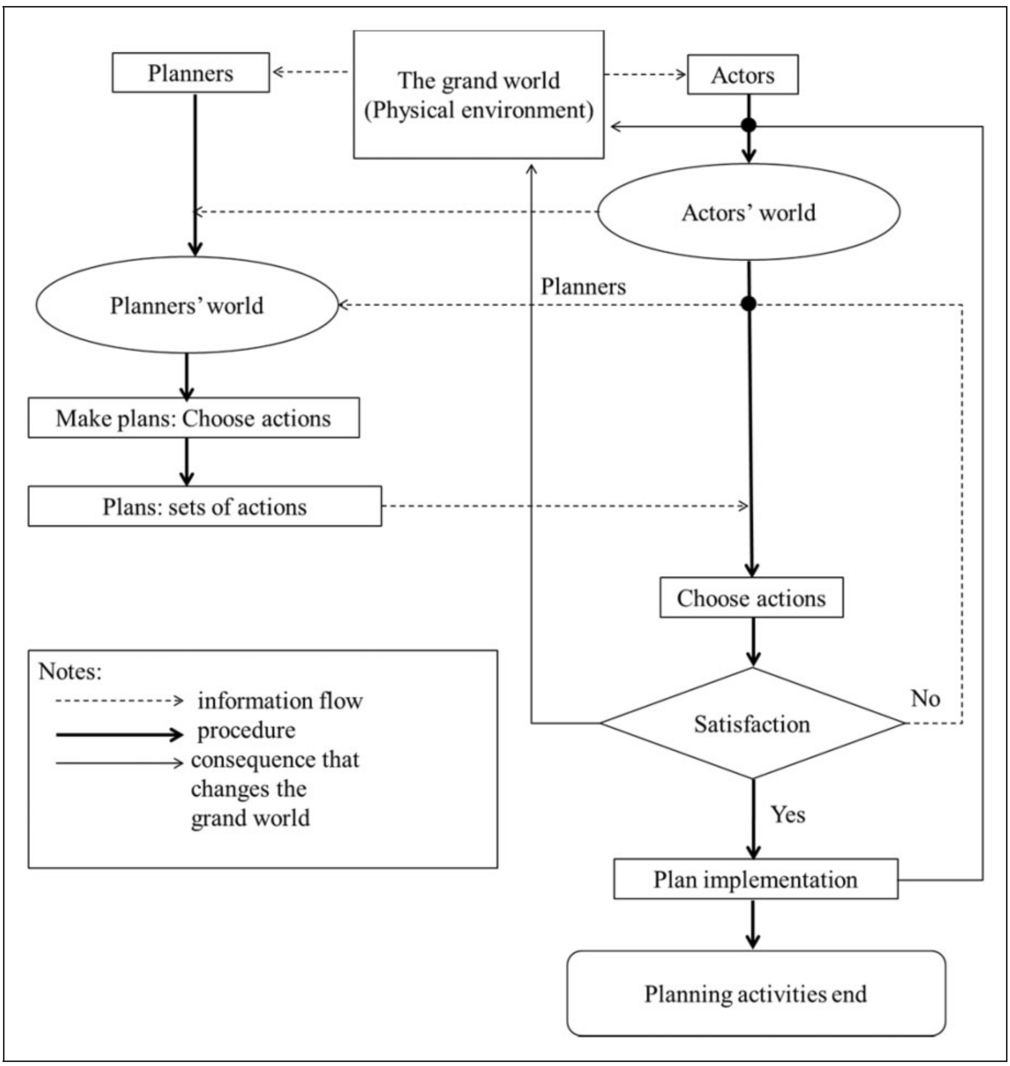

Figure 1. Conceptual framework of the planning procedure in each time interval.

process differs depending on the happenings of the plan revision due to the two reasons depicted. The planner must revise his/her plans until the planning horizon is reached, the resources depleted (time, labor, and money), or the agents are satisfied with the status quo when the planning process comes to a complete stop. Planning horizon identifies the scope in time the plan covers. For example, in a 20-year plan for urban growth boundaries with four 5-year time intervals, the 20 years are the planning horizon and the five years are the time interval.

In the first iteration of the planning process, the planner and the actor make observations in the grand world resulting in a set of states which are subsets of the states in the grand world. In addition to a set of states, the planner and the actor each, when faced with the planning problem, will figure out a set of actions (or strategies, LaValle, 1992) from which to select the optimal ones to maximize their expected utilities, respectively. This ensemble of sets of states and actions constitutes the planner's or the actor's world, all being small worlds. At this time, the actor will inform the planner of the ensemble of states and actions as planning situation; therefore, the planner's world incorporates actually partial planning situation of the actor's world. Procedurally speaking, the actor's world is formed before the planner's world. Once the planner's world has been constructed, he/she will make decisions in that world, that is, he/she will select the optimal action in order to maximize the planner's expected utility so that the standard of utility maximization can be used as a basis for choosing actions. The set of actions that maximize the planner's expected utilities in the m different time intervals constitute a plan which is a decision made by the planner. The planner then signals the plan to the actor to follow as information provision. The actor will then select actions based on the provided information. After the actor takes actions, consequences occur in the grand world which in turn change the perception in the planner's or the actor's world yielding new information. The new information might trigger off the changes in the decision world of which the actor will inform the planner, resulting in the changes in the planning world. The planner will then choose actions anew, that is, revise the plan, and at this time the second iteration commences of the planning process. When the planning horizon is reached, the available resources depleted, or the agents are satisfied with the status quo, the iterative procedure will stop. Note that no equilibrium is assumed in the iterative procedure.

Based on the iterative procedure depicted, we argue that planning is a dynamic process that constantly generates information and provides feedback, and the planner and the actor play an important role in the procedure. Revising and making plans are the same except that revising plans is caused by the changes in the planner's world resulting from different information structures that the actor provides. As a result, we focus on the communication between the planner and the actor in the first iteration of the procedure in order for the planner to make plans (sets of contingent actions) for the actor to follow, and as the actor is unsatisfied with the plan made by the planner (unable to maximize the actor's utility), how the actor would signal the planner to revise the plan. The information sent by the actor to the planner is the states and actions of his/her small world, while the information sent by the planner to the actor is a plan, also a set of actions. The decision made by the planner is to select plans, while the decision made by the actor is to choose actions. In the next section, we will formulate these ideas in a mathematical model as the main purpose of the present paper.

Interpretation of planning behavior

In the present paper, we attempt to formulate the basic ideas depicted in the previous section in terms of mathematical language, including the planner's and actor's worlds and the definition and process of plan making, implementation, and revision. The utilization of mathematical langue is to interpret explicitly normative planning behavior in order to derive fundamental hypotheses for future empirical research.

The planner's and the actor's worlds

In order to model a choice theory under uncertainty, the most convenient approach is first to understand the states of the world. States, denoted by S, describe fully a world and if the occurrence of the states of the world is certain, then the decision maker knows the consequence resulting from each action (Arrow, 1979). If the occurrence of the states is uncertain, probability and random variables are used to compute the expected values to cope with such uncertainty in order to evaluate actions. To generalize, we consider the case of uncertainty in the present paper.

The states of the grand world. The grand world is the objective, physical environment which can be described by q random variables forming a random vector. To simplify, assume that q is finite. The q random variables or random vector can be denoted by $S_v = (S_1, S_2, \ldots, S_q)$, in which we assume that S_i, $i = 1, 2, \ldots, q$, are mutually independent and exclusive and that S_i are discrete random variables of which the values are either 0 or 1. This is a rather strong assumption and implies that the grand world is discrete rather than continuous. Whether the physical world is discrete or continuous is a debatable question and subject to research on ontology, which is beyond the scope of the present paper. According to the two assumptions above, any phenomenon or consequence in the grand world can be described through the q random variables as the 2^q dimensional vector. For example, $S^1_{gw} = (1_{(1)}, 1_{(2)}, \ldots, 1_{(q)})$ is the first vector describing the grand world in which the subscripts i stand for the values of the ith random variable. We call S^1_{gw} a state in the grand world and we have 2^q states to describe the grand world. Assume that S_{gw} is the set of all possible states in the grand world, then $S_{gw} = \{S^1_{gw}, S^2_{gw}, \ldots, S^{2^q}_{gw}\}$ and there exists a probability distribution F_{gw} among the states and is known a priori.

The states in the planner's and the actor's worlds. After the planner and the actor make observations in the grand world, they form cognitively the planner's and the actor's subjective worlds. For example, because of the limitation of cognitive capability, if the planner only considers six random variables of S_8, S_9, S_{14}, S_{15}, S_{20}, and S_{21}, all other random variables being ignored, then there are $2^6 = 64$ six dimensional vectors of states describing the planner's world. One such vector is $S^1_p = \{1, 0, 0, 0, 0, 0\}$ among the $2^6 = 64$ possible vectors of states. For the same reason, assume that the actor considers four random variables, S_{21}, S_{22}, S_{27}, and S_{28} only; therefore, there are $2^4 = 16$ four dimensional vectors of states describing the actor's world. As a result, the planner's and actor's worlds are subsets of the states in the grand world and they do not necessarily coincide. The example above shows that only S_{21} is the random variable common to the planner's and the actor's world. Note that the selection of random variables is not random, but subject to the planner's and the actor's cognitive capabilities, preferences, and intents.

To generalize, assume that after the planner makes observations in the grand world and he/she comes up with p effective random variables, ignoring other q-p random variables, and the actor considers a random variables, ignoring other q-a random variables, where p and a are less than or equal to q and may or may not be equivalent. In this case, the planner's world is described by 2^p states of the grand world, whereas the actor's world is depicted by 2^a states of the grand world. Let S_p and S_a denote the planner's and the actor's worlds, respectively, and we have $S_p = \{S_p^1, S_p^2, \ldots, S_p^{2^p}\}$ and $S_a = \{S_a^1, S_a^2, \ldots, S_a^{2^a}\}$, where S_p^i, $i = 1, 2, \ldots, 2^p$, and S_a^i, $i = 1, 2, \ldots, 2^a$, are p and a dimensional vectors composed of 0's and 1's. For example, $S_p^1 = (1_{(1)}, 0_{(2)}, \ldots, 0_{(p)})$ and $S_a^1 = (1_{(1)}, 1_{(2)}, \ldots, 0_{(a)})$ represent the first states in the planner's and the actor's worlds, respectively, the subscripts i being the ith random variable. The elements of S_p^i include two properties: (1) they are mutually independent and exclusive, meaning that the probability that S_p^1 obtains does not affect the probability that S_p^i $(i \neq 1)$ obtains and vice versa and that only one element can occur and (2) the probability that S_p^i obtains is imposed subjectively by the planner. The two properties apply to the elements of S_a^i as well except that the probability that S_a^i obtains is given subjectively by the actor. In addition, $S_p \subseteq S_{gw}$ and $S_a \subseteq S_{gw}$.

The planner's and the actor's actions. Actions (or strategies, LaValle, 1992) are a function that transforms states to consequences, that is, with each state in the world there associates a consequence (Savage, 1954). According to this definition, assume that the actor knows the state of the world, and that he/she knows the consequence resulting from each action (Radner, 1979). In addition, if for each state in the world, two actions correspond to the same consequence, then the two actions can be treated as equivalent (Savage, 1954). This assumption is very strict, but the purpose is to simplify the modeling effort. It is very likely that the consequences resulting from actions may be uncertain and this case will be dealt with elsewhere in the future.

Plan is defined here as a set of contingent, related actions (Hopkins, 1980). According to this definition, before the planner makes plans, he/she will figure out all possible actions spanning the planning horizon within each time interval, or a plan, and the planner will select one among these plans.

Assume that A is the set of all possible actions within each time interval of the planning horizon and we define the action sets for the planner and the actor as follows:

Definition 1: Action set. Let $A_{t_i}^p = \{a_{ij}\}$ and $A_{t_i}^a = \{\bar{a}_{ij}\}$ denote the action sets for the planner and the actor respectively, representing all possible actions in different time intervals t_i, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, k$, where i symbolizes time intervals and there are m such intervals; j represents all possible actions in each time interval and there are k such actions. Note that m and k may be variable, but to simplify without loss of generality, they are fixed here as constants.

The formation of the planner's and the actor's worlds. According to Savage's definition, a small world is an ensemble of states and actions available for selection. We have defined the planner's and the actor's states and actions as shown in Section "The states in the planner's and the actor's worlds"; for convenient purposes, we first depict the actor's world, SW_a, as follows

$$SW_a = \left(S_a^i, A_{t_i}^a\right) = \left\{\left(s_a^i, \bar{a}_{ij}\right)\right\}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, k \tag{1}$$

In equation (1), SW_a is the actor's world; S_a^i describes the vector of states for the actor in time interval t_i; $\bar{a}_{ij}$ denotes the actions available to the actor in time interval t_i. After the decision world is formed, the actor will inform the planner of the small world, including a set of states and a set of actions; therefore, the planner's world includes a subset of the elements of the decision world. Let the planner's world be denoted by SW_p and we have

$$SW_p = \left({S^i}_p, {A^p}_{t_i}\right) + (\Delta S, \Delta A), \quad i = 1, 2, \ldots, m \tag{2}$$

In equation (2), SW_p denotes the planner's world; S_p^i stands for a set of states in the planner's world; ΔS represents the states in the actor's world that are not considered by the planner, that is, $\Delta S = S_a^i - S_p^i$; $A_{t_i}^p$ symbolizes the planner's action set; ΔA denotes the actor's actions in his/her action set, but not included in the planner's action set, that is, $\Delta A = A_{t_i}^a A_{t_i}^p$. Let $S_p^{i'} = S_p^i + \Delta S$ and $A_{t_i}^{p'} = A_{t_i}^p + \Delta A$, and we have

$$SW_p = \left(S_a^{i'}, A_{t_i}^{p'}\right) = \left\{\left(s_a^{i'}, a'_{ij}\right)\right\}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, k \tag{3}$$

where $s_p^{i'}$ and a'_{ij} represent the set of vectors of states and actions for the planner in time interval t_i.

Plan making, implementation, and revision

The expected utility theorem. Plan making, in this context, is equivalent to selecting actions for different time intervals in the planner's small world; therefore, after the small world is formed, the planner will choose the optimal action (plan) among a set of actions (plans) in the small world based on the subjective expected utility model. Utility is a real number function of consequences (Savage, 1954), denoted by U. In economic theory, utility indicates the satisfaction level of desire, and expected value can be calculated to measure the decision maker's preferences in relation to actions. The assumption of the decision maker's preferences in relation to actions implies that there exists a utility function, u, on consequences and a subjective probability function, Φ, so that the expected utility function U is defined as follows

$$U(a) = \sum_{j=1}^{n} u[a(s_j)]\Phi(s_j) \tag{4}$$

where n is the number of states.

Equation (4) represents the preferential priority of actions, where a denotes actions; s_j symbolizes a state; $a(s_j)$ represents the consequences resulting from actions, given that s_j obtains. Action a_1 is preferred to action a_2 if and only if $U(a_2) \leqq U(a_1)$. Equation (4) defines the expected utility function U as a positive, linear combination, thus a cardinal utility function, meaning that utility can be measured and calculated numerically in a concrete way. Based on the exposition above, we can derive the following expected utility theorem which serves as the basis to interpret how the planner selects actions, that is, making plans.

The expected utility theorem. For the states $s_p^{i'}$ in the planner's world for time interval t_i, there exists a probability measurement Φ, and for the consequences c_{ij}, there exists a real number function u, so that actions $a_{i2} \precsim a_{i1}$ if and only if $U(a_{i2}) \leq U(a_{i1})$, where

$$U(a_i) = \sum_{l=1}^{n} \{u(c_{ij})\} = \sum_{l=1}^{n} \{u[a_{ij}(s_l)]\} = \sum_{l=1}^{n} u[a_{ij}(s_l)]\Phi(s_l), \quad i = 1, 2, \ldots, k$$

Plan making. Plan making is equivalent to the planner's selecting an optimal action among those in his/her small world. According to equation (3), the planner's world includes an action set, $A_{t_i}^{p'}$, available for selection, and a set of states, $S_p^{i'}$, in time interval t_i. In the action set, there exists m time intervals in each of which there are $k+\Delta A$ actions available for selection, and in the set $S_p^{i'}$ there are $2^{(p\ +\Delta S)}$ states, where ΔA and ΔS are the actions and states in the actor's world not considered by the planner. Each action corresponding to a state in the world will result in a consequence, that is

$$a_i(s_j) = c_{ij}, \quad i = 1, 2 \ldots, k + \Delta A; \quad j = 1, 2 \ldots, 2^{(p+\Delta S)} \tag{5}$$

In each time interval, there thus exists $k' \times 2^{p''}$ consequences ($k' = k + \Delta A$, $p'' = p + \Delta S$) and we can denote the set of these consequences as C_p. For the set C_p, there exists a real number utility function $u(c_{ij})$ and a probability distribution φ_p on $S_p^{i'}$, and in a given time interval, the expected utility function for the k' actions is

$$U(a_i) = E\{u(a_i(s_j))\} = \sum_{j=1}^{2^{p''}} u[a_i(s_j)]\varphi_p(s_j), \quad i = 1, 2, \ldots, k' \tag{6}$$

According to equation (5), we have

$$U(a_i) = \sum_{j=1}^{2^{p''}} u(c_{ij})\varphi_p(s_j), \quad i = 1, 2 \ldots, k' \tag{7}$$

Note that equation (7) does not consider the discount factor of utility over time.

According to the expected utility theorem, the planner will select the action that maximizes the expected utility in a time interval. In other words, assume that the action $a_{t_1}^*$ yields the maximum expected utility $U(a_{t_1}^*)$ among k' actions in time interval 1, that is

$$U(a_{t_1}^*) = \max\{U(a_{11}), U(a_{12}), \ldots, U(a_{1k'})\} \tag{8}$$

where a_{1j}, $j = 1, 2, \ldots, k'$, are the actions available in time interval 1 and the planner prefers the action $a_{t_1}^*$ to other actions; therefore in the first time interval, the planner will choose $a_{t_1}^*$. By the same logic, the planner will come up with m actions that maximize expected utilities over the planning horizon and this set of optimal actions yielding the maximum expected utilities is a plan. Let P^* denote the set of such actions and a plan can be defined as below:

Definition 2: The plan. $P^* = \{a_{t_1}^*, a_{t_2}^*, \ldots, a_{t_m}^*\}$ is the plan made by the planner. It is a set of contingent, but *independent* actions, assuming that these actions are independent in that taking one action does not affect the consequences of other actions and thus whether other actions would be taken.

Plan implementation and revision. After the plan is made as shown in Definition 2, the planner's activity comes to an end. He/she will then signal the content of the plan to the actor who will act accordingly by selecting an action in each time interval. The decision made by the actor occurs in the decision world and when the actor receives the signal sent by the planner, the actor's action set has changed resulting in the decision world in time interval t_i as

$$SW'_a = \left(S_a^i,\ \{A_{t_i}^a \cup P^*\}\right) = \left(s_a^i,\ \{\bar{a}ij \cup a_{t_i}^*\}\right) \tag{9}$$

In equation (9), P^* is the content of the plan signaled by the planner and $a^*_{t_i}$ denotes the action that maximizes the planner's utility in each time interval t_i. If in the decision world the set of actions does not include the plan made by the planner, then in the decision world, there will be $k+1$ actions in each time interval available for selection (In the actor's action set, for each time interval there are originally k actions, plus the optimal action signaled by the planner; there are therefore totally $k+1$ actions). In the set S^i_a, there are totally 2^a states that describe the actor's world; as a result, the number of consequences is equal to $(k+1) \times 2^a$, denoted as C_a. There exists a probability distribution ϕ_a on S^i_a, and a utility function $u(c_{ij})$ on consequences. The selection criterion for the decision maker in relation to actions is also based on the expected utility theorem. The actor will come up with m actions for all time intervals to maximize his/her expected utilities.

Assume that the actor and the planner choose the same action; the action will be taken. If the actor is not satisfied with the plan made by the planner, that is, if the action selected by the actor is different from that suggested by the planner, then the actor will inform the planner of the difference. The planner will reevaluate the states in the actor's world, modify his/her own planning world, and reselect the optimal action, constituting plan revision. Plan revision is thus equivalent to the modification of the planner's world, and equation (3) becomes

$$SW'_p = \left(S^{i''}_p, A^{p''}_{t_i}\right) = \left(\left\{s^{i''}_p, a''_{ij}\right\}\right), \quad i = 1, 2, \ldots, m, \quad j = 1, 2 \ldots, k \tag{10}$$

In equation (10), SW'_p is the modified planning world, $S^{i''}_p$ is the planner's new understanding of the states in the planning world, $A^{p''}_{t_i}$ denotes the new action set resulting from the planner's renewed understanding of the planning problem. The iterative procedure will continue until the resources available (such as time, labor, and money) are exhausted, or the planning horizon is reached, or the agents are satisfied with the status quo and the planning activity will come to an end. Note again that no equilibrium is required in the iterative procedure.

Discussion

It is partially true that no one in the real world makes plans and acts accordingly in such an orderly way as depicted in the present paper because of complexity in real urban development, but the modeling practice clarifies the axiomatic logic of planning behavior that takes into account time and dynamic details explicitly For example, it is now well known that the principle of utility maximization does not hold in some situations, at least in the experimental settings (e.g. Kahneman, 2013). These empirical findings could be incorporated in the model, for example by replacing utility theory with prospect theory (Kahneman and Tversky, 1979). In addition, it is possible to test experimentally that whether the planner and the actor behave as depicted in the analytical model of the simplified planning environment. For example, are the planner's and the actor's worlds formed according to the definitions provided in Section "The states in the planner's and the actor's worlds"? Is the planning procedure shown in Figure 1 valid? Does the planner make plans according to Definition 2? Does the actor choose the optimal action according to the description in Section "Plan implementation and revision"? Besides further empirical validation, we could also examine some planning issues analytically based on the axiomatic planning model that would be of interest to planners. For example, we could verify the optimal planning scope in time, the most effective planning procedure, and the most efficient form of plans that accord with the decision world. We may expect that much deviation exists between what the model predicts and what actually happens, but by exploring into planning behavior empirically we may gain a better understanding of how planners and actors behave in the real world.

It is exposited that the plan made by the planner is a set of independent actions (Definition 2), but what matters in planning with complexity is making multiple, linked decisions (Han and Lai, 2011; Hopkins, 2001; Lai and Han, 2014). This is because in the model depicted here planning occurs in a simplified environment where the random variables under consideration are independent. In a world of complexity, such as cities, most, if not all, development decisions are interdependent, irreversible, indivisible, and imperfectly foreseeable (Hopkins, 2001). Therefore, in order for the model to be valid in reality, it must be so constructed to cope with the four I's.

The notion of small world is reminiscent of the idea of frame in decision theory. Elsewhere, we have shown that utility is frame dependent, in contrast to a universal utility function across different frames (Lai et al., 2017). The model depicted here as well as many other works based on the standard utility theory assume that utility is invariant. The idea of frame dependent utility could be incorporated in the behavioral model of planning in the future to take into account the more complex environment with multiple planners and actors. In addition, the axiomatic logic of planning proposed here could be automated in order to build planning models in planning support systems. For example, given a target of urban development, a set of behavioral rules based on the axiomatic logic could be "solved" so that planners and actors could work together for the desired urban development to emerge. Most of the extensions above of the model could be coded into computer simulations. For example, Lai (1994) provides a simulation framework that takes into account decisions, plans, actions, and consequences as depicted in the previous sections with additional consideration of group formation, preference evolution, and commitment level. Agent-based modeling techniques could be incorporated into such simulations to observe how plans interact with the city in a complex way.

The current formulation apparently does not take into account strategic behavior as exposited by game theory (e.g. Knaap et al., 1998); rather, it considers the planner and the actor as a team or coherent group. This assumption may limit the scope of application of the model, but we argue that in some planning situations, planners as manifested by the local government and actors as represented by developers might form coherent groups or coalitions to share plans to strive for common goals. For example, a regional planning agency (the planner) may conduct planning and share the resulting plans with local authorities (the actors) that benefit all participants.

Finally, the use of subjective expected utility in the model normally implies risk neutrality that fits the situations that take into account balance between expected costs and benefits without immense potential gains and very low probabilities. In order to incorporate more radical cases, the model needs to elaborate more on the assumptions of the shape of utility functions as convex or concave in relation to risk attitude other than risk neutrality, such as risk prone or risk averse.

Conclusions

Planning has long been perceived as intervention in a spatial system that tends toward equilibrium. In this perspective, time is implicit and dynamic details do not matter. As a result, little has been said in the literature about planning behavior that takes into account time and dynamic details. Elsewhere, we have proved axiomatically that plans matter for cities and concluded that urban complexity results from the four I's which can be dealt with through plans (Lai, 2018). As a result, exploration into planning behavior is important in the face of complexity in which path dependence is the rule rather than exception. [AQ2]As depicted in the 'Introduction' section, such an understanding can help predict the agents' behavior and design planning activities and procedure accordingly. The important idea in the present paper is: In the planning process of non-equilibrium systems, planning behavior and plans result from the interaction between the planner and the actor. Our axiomatic planning model is built on Savage's (1954) notion of small world, Hopkins's (2001) idea about planning and plans as information gathering and producing as well as a set of contingent, related actions, and Marschak's (1974) theory of teams, in order to formalize these conceptions through mathematical language into a set of explanations about normative planning behavior. Such a model is expected to serve as a starting point to explain observed planning behavior in reality as a basis for future empirical research.

Normative behavioral models are often accused by scholars of being far from reality (e.g. Dawes, 1988). We simplified the planning environment to explore into planning behavior without claiming to fully explain that behavior in the real world. Planning behavior in reality is much more complex because it includes multiple, interacting agents of which the complexity of the planning process is beyond what the simplified model can describe here. However, we proposed a theoretical framework which would help understand the agents' behavior, including planners, and provide a basis for future analytical and empirical works, in the hope of initiating empirical hypotheses for testing and exploring some basic planning issues, such as planning rationality, planning scope, and useful planning procedures. Much remains to be done.

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